Theory of games: Introduction and basic terminology, pure strategy games (including identification of saddle point and value of the game), Principle of dominance, mixed strategy games (only arithmetic method for 2 x 2 games)

The term 'game' represents a conflict between two or more parties. There can be several types of games, e.g. two-person and n-person games, zero-sum and non-zero-sum games, constant-sum games, co-operative and non-co-operative games, pure strategy games and mixed strategy games, etc.

When there are two competitors playing a game, it is called a two-person game. If the number of competitors are N (N > 2), it is known as an N person game. When the sum of amounts won by all winners is equal to the sum of the amounts lost by all losers, we call it a zero-sum game. In a non-zero-game there exists a jointly preferred outcome. In other words, in a zero-game or a constant-sum game the sum of gains and losses of the game is zero. As opposed to this, if the sum of gains or losses is not equal to zero, we call it a non-zero-sum game. When the best strategy for each player is to play one particular strategy throughout the game, it is known as a pure strategy game. In case the optimum plan for each player is to employ different strategies at different times, it is called a mixed strategy game. When there is communication between the participants they may reach agreement and increase their pay-off through some forms of co-operative game, otherwise it is a non-co-operative game.

Game theory is the formal study of conflict and cooperation. Game theoretic concepts apply whenever the actions of several agents are interdependent. These agents may be individuals, groups, firms, or any combination of these. The concepts of game theory provide a language to formulate structure, analyze and understand strategic scenarios.

Rules of the Game

Game theory is applicable to situations that satisfy the following conditions:

- The number of competitors is finite.
- The players act rationally and intelligently.
- Each player has available to him a finite set of possible courses of action.

- There is a conflict of interests between the participants.
- The players make individual decisions without directly communicating.
- The rules governing the choice are specified and known to the players.
- The players simultaneously select their respective courses of action.
- The payoff (outcome) is fixed and determined in advance

Basic Terminology

1. Strategy

A strategy for a player has been defined as a set of rules or alternative courses of action available to him in advance, by which player decides the courses of action that he should adopt. There are two types:

a. Pure Strategy

If the player selects the same strategy each time, then it is a pure strategy. In this case each player knows exactly what the other is going to do, i.e. there is a deterministic situation and the objective of the players is to maximize gains or to minimize losses.

b. Mixed Strategy

When the players use a combination of strategies and each player is always kept guessing as to which course of action is to be selected by the other, then it is known as a mixed strategy. Thus, there is a probabilistic situation and the objective of the player is to maximize expected gains or to minimize losses. Thus, mixed strategy is a selection among pure strategies with fixed possibilities.

2. Optimal Strategy

A course of action which puts the player in the most preferred position irrespective of the strategy of his competitors. Any deviation from this strategy results in a decreased pay-off for the player.

3. Value of the Game

The expected pay-off of the game when all the players of the game follow their optimum strategies. The game is called fair if the value of the game is zero and unfair if it is non-zero.

4. Two-person zero-sum game

There are two types of Two-person zero-sum games. In one, the most preferred position is achieved by adopting a single strategy and therefore the game is known as the pure strategy game. The second type requires the adoption by both players of a combination of different strategies in order to achieve the most preferred position and is, therefore, referred to as the mixed strategy game.

5. Pay-off matrix

A two-person zero-sum game is conveniently represented by a matrix. The matrix which shows the outcome of the game as the players select their particular strategies, is known as the pay-off matrix. It is important to assume that each player knows not only his own list of possible courses of action but also that of his opponent.

Let player A have m courses of action (A1, A2, A3, …, Am) and player B have n courses of action (B1, B2, B3, …, Bn). The numbers m and n need not to be equal. The possible number of outcomes is therefore (mxn). These outcomes are shown in the following matrices:

A's pay-off matrix

		Player B		
		B1	B2 . . .	Bn
	A1	a11	a12	a1n
Player A	A2	a21	a22	a2n
	.	.	.	.
	.	.	.	.
	.	.	.	.
	Am	am1	am2	amn

B's pay-off matrix

		Player B		
		B1	B2 . .	Bn
	A1	-a11	-a12	-a1n
Player A	A2	-a21	-a22	-a2n
	.	.	.	.
	.	.	.	.
	.	.	.	.
	Am	-am1	-am2	-amn

PURE STRATEGIES- GAME WITH SADDLE POINT

Consider the pay off matrix of a game which represents pay off of player A. Now, the objective of the study is to know how these players must elect their respective strategies so that they may optimise their pay off. Such a decision making criterion is referred to as the minimax-maxmin principle.

The MAXIMIN-MINIMAX principle

- Maximin Criteria- The maximising player lists his minimum gains from each strategy and selects the strategy which gives the maximum out of these minimum gains
- Minimax Criteria- The minimising player lists his maximum loss from each strategy and selects the strategy which gives him the minimum loss out of these maximum losses

Some definitions-

Saddle point (equilibrium point)- it is position in the pay off matrix where maximum of row minima coincides with the minimum of the column maxima. The saddle point need not be unique. Let v1= maximin value of the game and v2= minimax value of the game. At saddle point v1=v2=v

The following steps are required to find out Saddle point

- Select the minimum value of each row & put a circle around it.
- Select the maximum value of each column and put square around it
- The value with both circle and square is the saddle point.

Value of game-the value of the game is the maximum guaranteed gain to the maximising player if both the players use their best strategy. The pay off the saddle point is called the value of the game denoted by v.

Fair game- game is called fair game if neither player has an advantage over the other. So v1=0=v2

Strictly determinable game- A game is said to be strictly determinable if v1=v=v2

Points to remember

- Saddle point may or may not exist in a given game
- There may be more than on saddle point then there will be more than one solution
- The value of game may be positive or negative
- The value of game may be zero which means fair game

PRINCIPLE OF DOMINANCE

Sometimes, it is observed that one of the pure strategies of either player is always inferior to at least one of the remaining ones. The superior strategies are said to dominate the inferior ones. Clearly, a player would have no incentive to use inferior strategies which are dominated by the superior ones. In such case of dominance, we can reduce the size of the payoff matrix by deleting those strategies which are dominated by the others.

General rules for dominance are

- If all the elements of a row, say kth row, are less than or equal to the corresponding elements of any other row, say rth, then kth row is dominated by rth row
- If all the elements of a column, say kth are greater than or equal to the corresponding elements of any other column, say rth, then kth column is dominated by the rth column
- Dominated rows or columns (kth row and kth column) may be deleted to reduce the size of payoff matrix, as the optimal strategies will remain unaffected.

Mixed strategy games (only arithmetic method for 2 x 2 games)

For a zero-sum two-person game in which each of the players, say A and B has strategies A1& A2and B1& B2respectively and the pay-offs as given below, then if p1is the probability with which A chooses strategy A1and if q1is the probability that B plays strategy B1, the pay-off matrix for player A is given by

		PLAYER B	
		B1	*B2*
PLAYER A	*A1*	*.a11*	*.a12*
	A2	*.a21*	*.a22*

The following formulae are used to find the value of the game and the optimum strategies.

p1= (a22- a21)/((a11 + a22) –(a21+a12)) ; p2= 1-p1

q1= (a22-a12)/ ((a11 + a22) –(a21+a12)) ; q2= 1-q1

V =(a11 a22-a21a12)/ ((a11 + a22) –(a21+a12))

Advantages of Game Theory

1. Game theory gives insight into several less known aspects, which arise in situation of conflicting interest.
2. Game theory develops a framework for analysing decisions making in such situation where inter-dependence of firm is considered
3. At least in two person zero games, game theory outlines a scientific quantitative techniques that can be used by players to arrive at an optimal strategy

Limitation of Game Theory

1. The assumption that players have the knowledge about their own pay off and pay offs of there is not practical.
2. The Techniques of solving gains involving mixed strategies particularly in case of large pay off matrix is very complicated.
3. All the competitive problems cannot be analyzed with the help of game theory