

# Lecture notes- (QTM) Decision Theory

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## DECISION THEORY-

*Decision theory : Introduction to risk and uncertainty, Decisions under Uncertainty using Laplace, maximin, Minimax, maximax, minimin, hurwicz and Savage Methods*

Some elements are common for all kinds of decisions

- The decision maker- the decision maker is refers to an individual or a group of individuals responsible for making a choice of an appropriate course of action amongst the available course of action
- State of nature- Consequences (or events) of any course of action are dependent upon certain factors beyond the control of the decision maker. These factors are called states of nature.

### Decision making environment

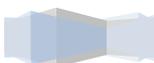
Decision analysis is used to determine optimum strategies where a decision maker is faced with several decisions alternatives. We may come across several decision making situations

- Decision under certainty
- Decisions under conflict
- Decisions under uncertainty
- Decisions under risk

### **Decisions under uncertainty**

Under conditions of uncertainty, only payoffs are known and nothing is known about the likelihood of each state of nature. Different persons have suggested several decision rules for making a decision under such situations

- Criterion of pessimism (Maximin or Minimax )
- Criterion of optimism (Maximax or Minimin)
- Laplace criterion



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- Savage
- Hurwicz

### Criterion of pessimism (Maximin or Minimax)

This criteria is based on the conservative approach to assume that, the worst possible is going to happen. The decision maker considers each strategy and locates the minimum pay off for each and then selects that alternative which maximise the minimum pay off. Thus, this criterion involves two steps.

Step I Find the minimum assured pay off for each alternative (course of action)

Step II Choose that alternative which corresponds to the maximum of the above minimum pay off

When dealing with the cost, the maximum cost associated with each other alternative is considered and the alternative that minimise this maximum cost is chosen. This is known as minimax criterion and it involves two steps.

Step I Determine the maximum possible cost for each alternative.

Step II Choose that alternative which corresponds to the minimum of the above costs.

### Criterion of optimism (Maximax or minimin )

This criterion is based on extreme optimism. In this criterion, the decision maker ensures that he should not miss the opportunity to achieve the greatest possible pay off or lowest possible cost. In maximax criterion, the decision maker selects that particular strategy which corresponds to the maximum pay off for each strategy. Thus, the maximax criterion consists of the following two steps

Step I Determine the maximum possible way off for each alternative.

Step II Select that alternative which corresponds to the maximum of the above minimum pay offs

In the decision problems dealing with costs, the minimum for each alternative is considered and then the alternative which minimizes the above minimum cost is selected. This is termed as minimin principle.



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## Laplace Criterion (equally likely decision criterion)

This criterion is based on, what is known as the principle of insufficient reason. Since the probabilities associated with the occurrence of various events are unknown, there is not enough information to conclude that these probabilities will be different. Hence it is assumed that all states of nature will occur with equal probability. That is, each state of nature is assigned an equal probability. As states of nature of mutually exclusive and collectively exhaustive, the probability of each of these must be  $1/(\text{number of states of nature})$

This criterion involves following steps

Step I Assign equal probabilities  $1/(\text{number of states of nature})$  to each pay off a strategy.

Step II Determine the expected pay off value for each alternative

Step III Select that alternative which corresponds to the maximum (and minimum for cost) of the above expected expected pay offs.

## Hurwicz criterion (Criterion of Realism)

This criterion suggests that a rational decision maker should neither be completely optimistic nor be pessimistic and therefore, must display a mixture of both. Hurwicz, who suggests this criterion, introduced the idea of a coefficient of optimism (denoted by  $\alpha$ ) to measure the decision maker's degree of optimism. This coefficient lies between 0 and 1, where 0 represents a completely pessimistic attitude about the future and 1, completely optimistic attitude about the future. Thus, if it is the coefficient of optimism, then  $(1 - \alpha)$  will represent the coefficient of pessimism. The working procedure is –

- Decide the coefficient of optimism  $\alpha$  and then the coefficient of pessimism  $(1 - \alpha)$
- Determine the maximum as well as minimum pay off for each, alternative and obtain the quantities  $h = \alpha (\text{maximum of each alternative}) + (1 - \alpha) (\text{minimum of each alternative})$
- Select an alternative with value of  $h$  as maximum.

For  $\alpha=1$ , the Hurwicz criteria is equal to the maximin or minimax criteria. For  $\alpha = 0$ , it is equal to maximax or minimin criteria.

A difficulty with this criteria is the appropriate selection of  $\alpha$  between 0 and 1.



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### Savage Criterion (criterion of regret, minimax regret criterion, opportunity loss decision criterion)

While the above criteria do not take into account the cost of opportunity losses by making the wrong decision, the Savage criterion does so.

The Savage criterion is based on the concept of regret (or opportunity loss) and calls for selecting the course of action that minimises the maximum regret. This criterion assumes that a decision maker feels regret after adopting a wrong course of action (alternative) resulting in an opportunity loss of pay off.

Step I From the given pay off matrix, develop an opportunity loss (or regret) matrix

(a) Find the best pay off corresponding to each state of nature (maximum for profit and minimum for cost)

(b)  $i$ th regret = (maximum pay off –  $i$ th pay off) for  $j$ th event if the pay offs represent profits  
= (minimum pay off –  $i$ th pay off) for  $j$ th event if the pay offs represent costs

Step II Determine the maximum regret amount for each alternative.

Step III Choose that alternative which corresponds to the minimum regrets

