Network

A network is a graphic representation of a project's operations and a composed of activities and events that must be completed to reach the end objective of a project, showing the planning sequence of time accomplishment, their dependence and inter-relationship.

The basic components of a network are

Activity- An activity is a task, or item of work to be done, that consume time, effort, money or other resources. An activity is represented by an arrow with its head indicating the sequence in which the events are to occur.

Event- An event represents the start (beginning) or completion (end) of some activity and as such it consume no time. It has no time duration and does not consume any resources. It is also known as a node. An event is generally represented on the network by a circle.

The activity can be further classified into the following three categories

1. Predecessor activity- An activity which must be completed before one or more other acj'tivities start is known as predecessor activity
2. Successor activity- An activity which started immediately after one or more of other activities are completed is known as successor activity.
3. Dummy activity- An activity which does not consume either any resource or time is known as dummy activity. A dummy activity is depicted by dotted line in the network diagram.

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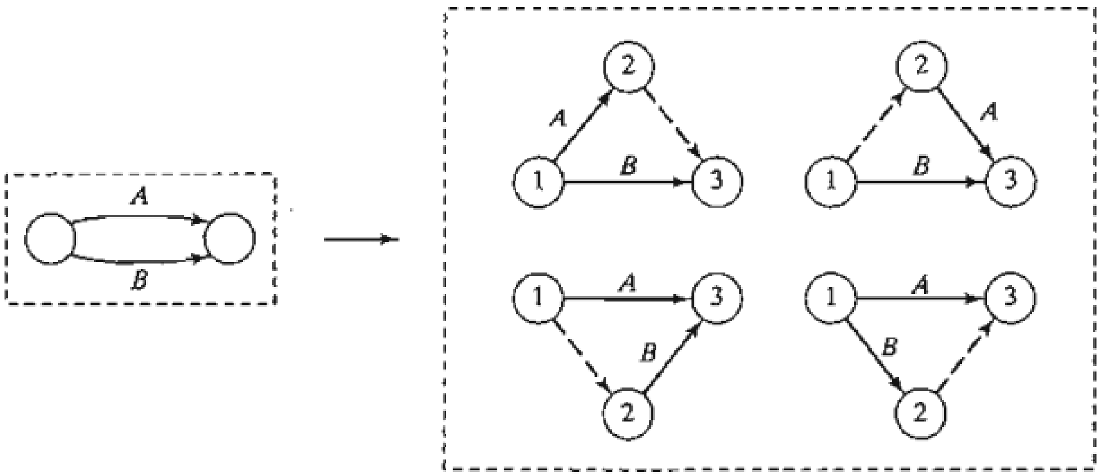
Use of dummy activity to produce unique representation of concurrent activities



Various known techniques are available for network planning and actions. We will discuss only CPM and PERT. These two methods provide a methodology for planning, scheduling and controlling of project implementation.

Prior to the development of PERT and CPM, the most popular technique for project scheduling was the bar or Gantt Chart. These charts show a graphic representation of work on a time scale. The primary limitation of this technique is its inability to show the inter relationships and interdependencies among the many activities which control the proegress of the project. Although it is possible to redraw the chart to show the inter-relationships but the confusion aries as the size of the project increases. To overcome such limitations PERT and CPM were proposed in the late 1950s.

PERT and CPM developed independently out of research studies that were being conducted by the US navy and DuPont company. PERT was an outgrowth of the US navy Polaris submarine missile program where as CPM was developed for planning the construction of chemical plants.

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Logical sequences in Network diagram

All the projects consist of certain activities that can begin only after certain others are completed. In fact, the entire project may be considered as a series of activities which may begin only after another activity or activities are completed. In a network schedule, these types of relationship are called constraints and are represented by inequalities.

In logical sequencing, following two types of errors are most common while drawing a network diagram. (See Book)

1. Looping
2. Dangling- No activity should end without being joined to the end event. If it is not so, a dummy activity is introduced in order to maintain the continuity of the system. Such end event other than the end of the project as a whole are called dangling events.

Rules for Network Representation

Three rules are available for constructing the network

1. Each activity is represented by one, and only one arrow (arc)
2. Each activity must be identified by two distinct end nodes & no two or more activities can have the same tail.
3. To maintain the correct precedence relationships, the following questions must be answered as each is added to network:
   (a) What activities must immediately precede the current activity?
   (b) What activities must follow the current activity?
   (c) What activities must occur concurrently with the current activity?
   The answer of these questions may require the use of dummy activities to ensure correct precedences among the activities.

Numbering the events- Fulkerson Rule

After the network is drawn in a logical sequence, every event is assigned a number. The number sequence must be such as to reflect the flow of the network. In event numbering, the following rules should be observed, which is also known as Fulkerson's rule.

(a) Event numbers should be unique
(b) Event numbering should be carried out on a sequential basis from left to right
(c) The initial event which has all outgoing arrows with no incoming arrow is numbered 0 or 1
(d) The head of an arrow should always bear a number higher than the one assigned at the tail of the arrow
(e) Gaps should be left in the sequences of event numbering to accommodate subsequent inclusion of activities, if necessary.

Objective for Network Analysis

1. Minimisation of total project cost
2. Minimisation of total project duration
3. Trade off between time and cost of project
4. Minimisation of idle resource
5. To minimise production delays

CPM/PERT

CPM/PERT are network based models designed to assist in the planning, scheduling and control of projects.

Project- A project is defined as a collection of interrelated activities with each activity consuming time and resources

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The objective of CPM/PERT is to provide analytic means for scheduling the activities. Followings are the steps of the techniques

1. We define the activities of the project, their precedence relationship and their time requirements.
2. The precedence relationship among the activities are represented by a network
3. Specific computations to develop the time schedule for the project. During the actual execution of the project things may not proceed as planned, as some of the activities may be expedited or delayed. When this happens, the schedule must be revised to reflect the realities on the ground. This is the reason for including a feedback loop between the time schedule phase and the network phase, as shown in following diagram.

6.5 CPM and PERT

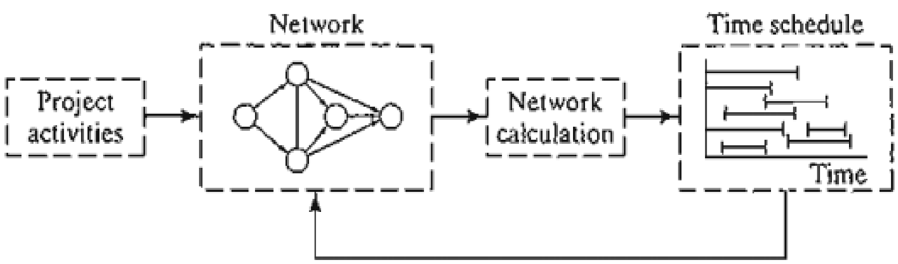

FIGURE 6.38
Phases for project planning with CPM-PERT

The two techniques, CPM and PERT, which were developed independently, differ in that CPM assumes deterministic activity duration and PERT assumes probabilistic durations.

CPM- It is commonly used for those projects which are repetitive in nature & where one has prior experience of handling similar projects. It is a deterministic model and places emphasis on time & cost for activities of a project.

PERT (Program evaluation & review Technique)- it is generally used for those projects where time required to complete various activities are not known as a priori. It is probabilistic model & is primarily concerned for evaluation of time. It is event oriented.

Before detail discussion of both methods, it is interesting to note their differences.

Difference between PERT and CPM

The difference between PERT and CPM arose primarily because of the original job for which the particular technique was developed. Initially the PERT technique was applied to research and development projects while the CPM was used towards construction projects. Both of them in common the notion of a critical path and are based on the network plan that determines the most critical activities to be controlled so as to meet completion dates.

The major differences between PERT and CPM are summarized as given below.

PERT	CPM
• Event oriented	• Activity oriented
• Probabilistic in nature	• Deterministic in nature
• Concerned with time	• Concerned with time and cost
• Used for new projects	• Used for repetitive projects

Critical Path Method (CPM)

The end result of CPM is the construction of the time schedule for the project. To achieve this objective conveniently, we carry out special computations that produce the following information,

1. Total duration needed to complete the project
2. Classification of the activities of the project as critical and noncritical.

An activity is said to be critical if there is no "leeway" in determining its start and finish times. A noncritical activity allows some scheduling slack, so that the start time of the activity can be advanced or delayed within limits without affecting the completion date of the entire project.

To carry out the necessary computations, we define an event as a point in time at which activities are terminated and others are started. In terms of the network, an event corresponds to a node. Define

E_i = Earliest occurrence time of event i

L_j = Latest occurrence time of event j

D_{ij}= duration of activity (i, j)

The definitions of the earliest and latest occurrence of event j are specified relative to the start and completion dates of the entire project.

The critical path calculation involve two passes: the **forward pass** determines the earliest occurrence times of the events, and the **backward pass** calculates their latest occurrences times.

Forward Pass (Earliest occurrence times, E)

Initial Step: Set E1 = 0 to indicate that the project starts at time 0

General Step: Given that nodes p, q, ….. and v are linked directly to node j by incoming activities (p, j), (q, j),… and (v, j) and that the earliest occurrence times of events (nodes) p, q, ….., and v have already been computed, then the earliest occurrence time of event j is computed as

$$E_j = \text{Max}\,(E_p + D_{pj}, E_q + D_{qj}, \ldots\ldots\ldots\ldots, E_v + D_{vj})$$

The forward pass is complete when En at node n has been computed. By definition Ej represents the longest path (duration) to node j.

Backward Pass (Latest occurrence times, L)

Initial step: Set Ln= En to indicate that the earliest and latest occurrences of the last node of the project are the same

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General Step: Given that nodes p, q, ….. and v are linked directly to node j by outgoing activities (p, j), (q, j),… and (v, j) and that the latest occurrence times of events (nodes) p, q, ….., and v have already been computed, then the latest occurrence time of event j is computed as

$$L_j = \text{Min} [L_p - D_{jp}, L_q - D_{jq}, \ldots\ldots\ldots\ldots, L_v - D_{jv}]$$

The backward pass is complete when L_1 at node 1 is computed. At this point $L_1=E_1=0$

Based on the preceding calculation, an activity (i,j) will be critical if it satisfies three conditions.

1. $L_i=E_i$
2. $L_j=E_j$
3. $L_j - L_i = E_j - E_i = D_{ij}$

The three conditions state that the earliest and latest occurrence times of end nodes i and j are equal and the duration Dij fits tightly in the specified time span. An activity that does not satisfies all three conditions is thus noncritical.

Numerical Example ------ in class

FLOAT

In case of non-critical activities, certain amount of spare time is available and this spare time is called “float”. There are three type of float

1. Total float
2. Free float
3. Independent float

Total float- it is defined as “the amount of time by which completion of an activity could be delayed beyond the earliest expected completion time without affecting the overall project duration time.

$TF_{ij} = L_j - E_i - D_{ij}$

Free float- In calculating the total float only a particular activity has been considered with respect to tail and head event times. But it may be necessary to find out how much an activity can float without affecting the flexibility of movement of the immediate succeeding activity. The free float is that part of the total float which can be used without affecting the float of the succeeding activities.

$$FF_{ij} = E_j - E_i - D_{ij}$$

Independent float – the independent float time of an activity is the amount of float time which can be used without affecting either the predecessor or the successor activities.

$$IF_{ij} = E_j - L_i - D_{ij}$$

The relation between Total, Free and Independent float is following-

$$IF_{ij} \leq FF_{ij} \leq TF_{ij}$$

Numerical Example------in class

PROGRAMME EVALUATION AND REVIEW TECHNIQUE (PERT)

In the CPM model, scheduling times of various activities, the critical path, and the project length were all determined on the basis of activity times which were assumed to be known and constant. But according to PERT originators, any given activity delineated in a network is unlikely to be completed on time. Sometimes all aspects of a job may be easier to complete than expected, while at other times, unexpected snags may occur, causing unplanned delays. This method uses three time estimates for an activity, rather than a single estimate. They are:

a. Optimistic time (a)- optimistic time is the duration of any activity when everything goes on well during the project.
b. Most likely time (m)- it is duration of an activity when some things go on well and some things go wrong while doing a project

c. Pessimistic time (b)- it is duration of an activity when almost everything goes against our will and lot of difficulties are faced while doing a project.

Assumptions of PERT

1. The activity durations are independent. That is, the time requirement to complete an activity will have no bearing on the completion time of any activity of the project.
2. The activity durations follows β distribution. B distribution is a probability distribution with density function

   $F(t) = K(t-a)^{\alpha} (b-t)^{\beta}$

   With mean $t_e = (a+4m+b)/6$ and standard deviation $(\sigma)=(b-a)/6$.

PERT PROCEDURE

- Step I Draw the project network
- Step II compute the expected duration of each activity te
- Step III compute the expected variance of each activity
- Step IV compute the earliest start, earliest finish, latest start, latest finish and total float of each activity.
- Step V determine the critical path and identify critical activities.
- Step VI compute the expected variance of the project length, which is sum of variance of all the critical activities.
- Step VII compute the expected standard deviation of the project length and calculate the standard normal deviation, which is

  $(Ts- Te)/ \sigma_T$

  Where Ts = specified or scheduled time to complete the project

  Te= Normal expected duration (duration of the project)

  σ_T = expected standard deviation of the project length.

Important:- If there are two or more critical paths in a given network, then the one with the largest variance value should be used for determining Te and σ_T.

In PERT, we calculate the expected time of each activity by using three time estimates-a, m and b. By using this estimated expected time for each activity, we can determine the critical path. By assumption, all activities are independent so we can use central limit theorem to represent one important feature of PERT- probability of meeting the schedule time.

<u>The Central Limit Theorem-</u> it states that the sum of several independent activity duration will tend to be normally distributed, with a mean equal to the sum of their individual job times and variance equal to the sum of their individual activity variance.

For a given project, if the critical activities are 1,2,...,k, we have

Te = Te1 + Te2 +... +Tek , Te= Normal expected duration (duration of the project) or critical path,

V_T = V1 + V2 +... +VK , V_T = Variance of the project length.

So probability distribution of times of completing an event can be approximated by the normal distribution.

<u>Normal Distribution-</u> If X is a continuous random variable following normal probability distribution with mean μ and standard deviation σ, then its probability density function (p.d.f) is given by

$$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Where π and e are the constant given by π = 22/7 and e = 2.71828 (the base of the system of natural logarithm)

The mean μ and standard deviation σ are called the parameters of the normal distribution.

$Z= (X- \mu)/ \sigma$ standard normal variable (S.N.V.)

With mean = 0 and variance= 1

Probability of completing the project on or before a specified time

On the basis of normal distribution, the probability of obtaining any specified date can be easily derived. For this, calculate the standard normal deviation,

$Z = (Ts- Te)/ V_T$

Ts= Specified or scheduled time to complete the project

The probability of completing the project by scheduling time (Ts) is given by

Prob $(Z < (Ts- Te)/ V_T)$

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Time and cost Analysis

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